Universität Zürich[UZH] | MethLab Journal Club

«Superlets»

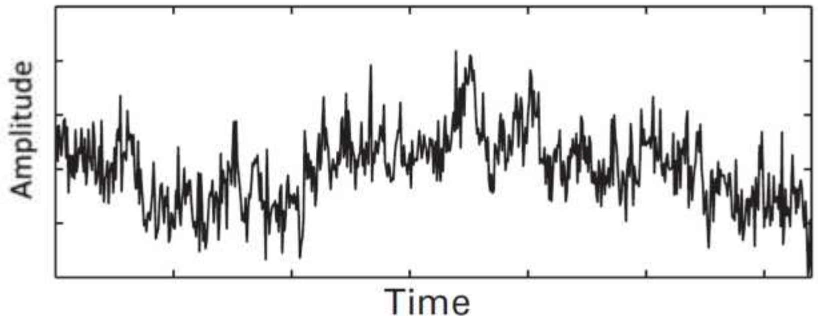
Time-frequency super-resolution with superlets

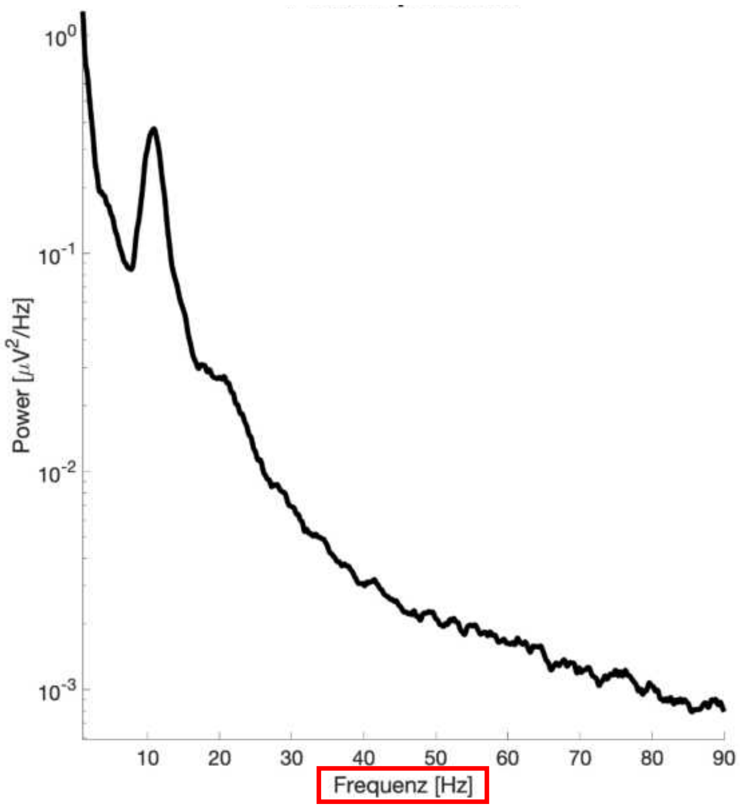
Moca et al., 2021

Analyzing Neural Time Series

Brain signals are **continuous**, **oscillatory**, and **noisy**
- Analyses with e.g. ERPs (averaging over segments of continous data)
- Other interest: how signal energy (power) is distributed across frequencies

→ Fourier Transform
- Decomposes continous signal into sine/cosine waves
- Produces a power spectrum (loss of temporal info!)

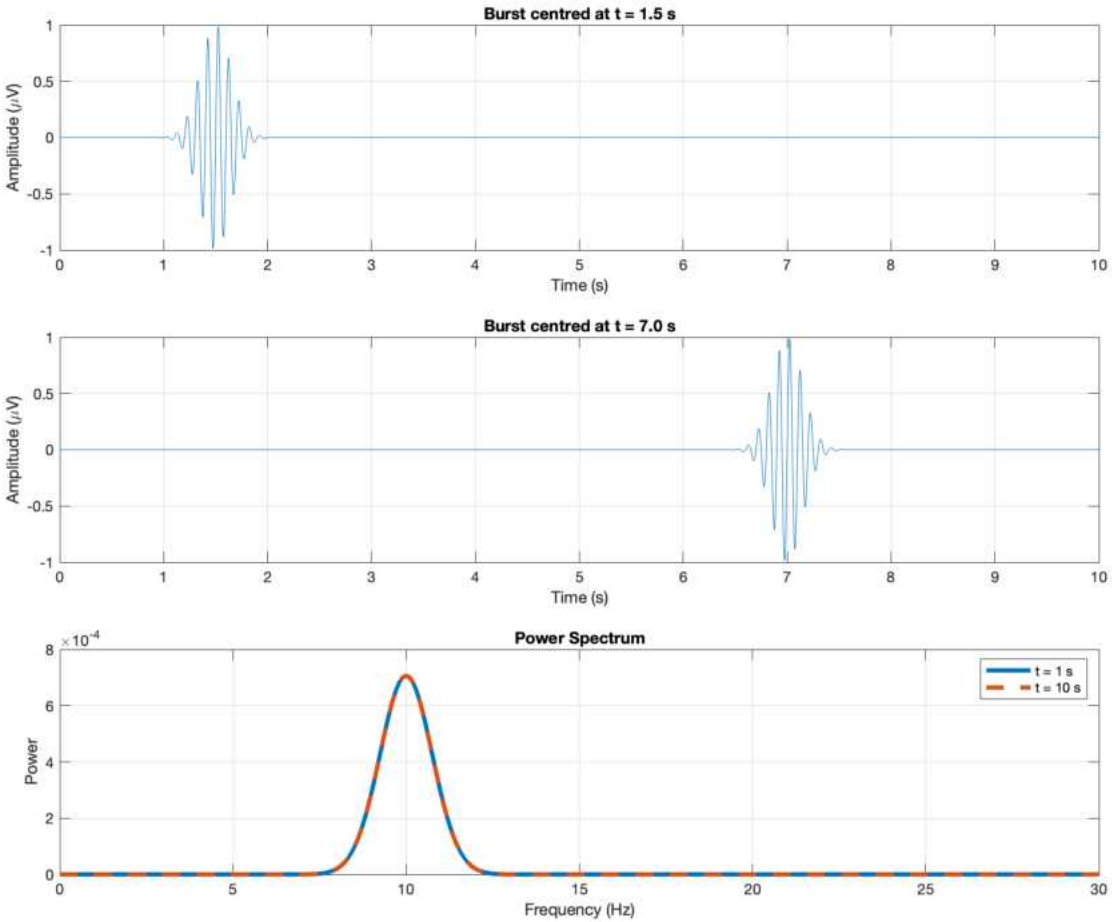




Problems with Fourier Transform

Loss oftemporal information
- Temporal development of neural activity
- When bursts occur

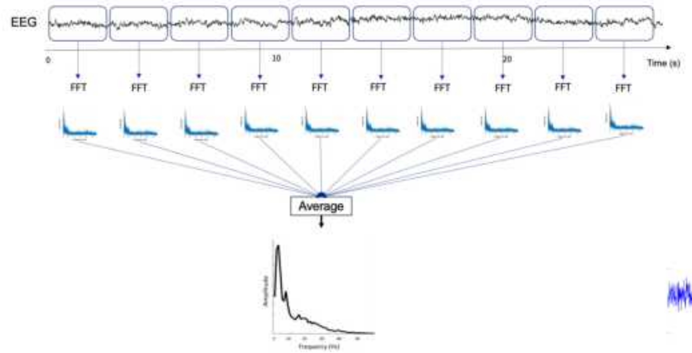
Assumes stationarity
- No change in properties of signal over time
- not ideal for time-varying brain activity



Solutions: P-Welch, STFT, Wavelets & Hilbert

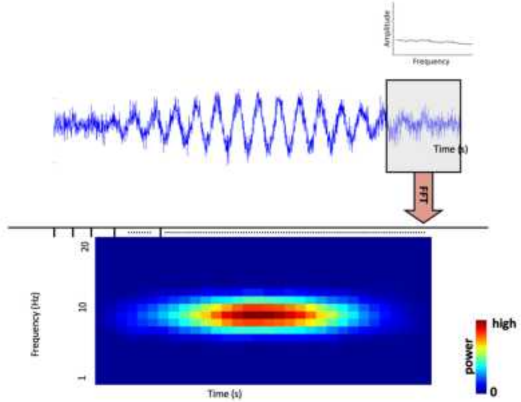
Welch's Method (P-Welch)

– Averages multiple **windowed segments**.
– Reduces variance in power spectral estimates.
– Solution to noisy and unstable single-trial Fourier spectra.
– NO TEMPORAL INFO!



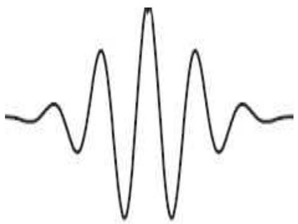
Short-Time Fourier Transform (STFT)

– Applies Fourier transform in **sliding windows**.
– Provides time–frequency representation with fixed resolution.
– Solution to lack of temporal information in standard Fourier.



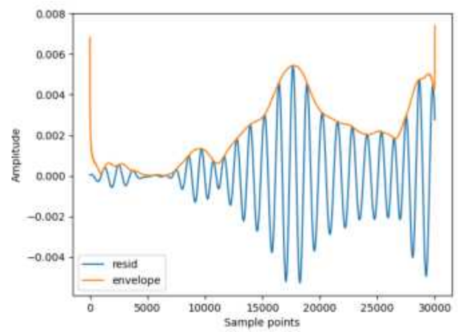
Wavelet Transform

– Uses **variable-length windows** (short for high freq, long for low freq).
– Provides multiresolution time–frequency analysis.
– Solution to trade-off in STFT between temporal and spectral resolution.



Hilbert Transform (after band-pass filtering)

– Extracts **instantaneous amplitude** and phase of a narrowband signal.
– Captures oscillatory dynamics trial-by-trial.
– Solution to Fourier's limitation in tracking non-stationary phase/amplitude dynamics.



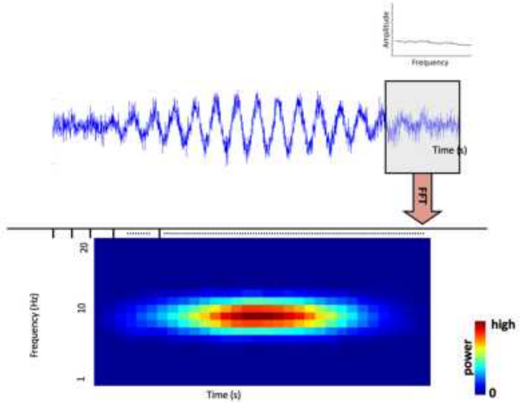
Solutions: Short-Time Fourier Transform (STFT)

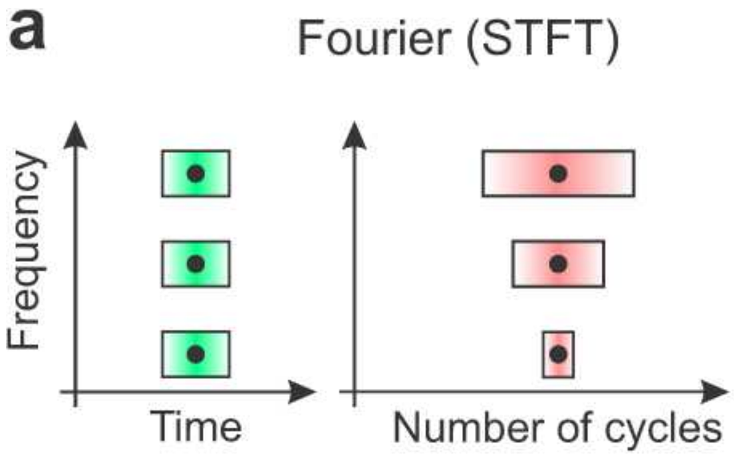
Applies Fourier transform in **sliding windows**

Provides time–frequency representation with **fixed resolution**

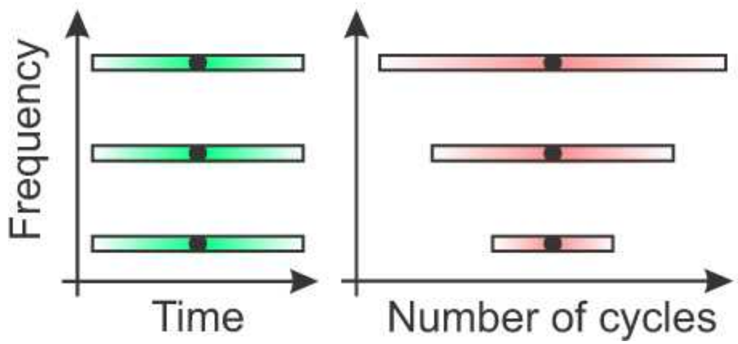
Trade-off: length of sliding window
- short windows: better temporal resolution, poor frequency resolution
- long windows: worse temporal resolution, better frequency resolution

→ Depends on research question





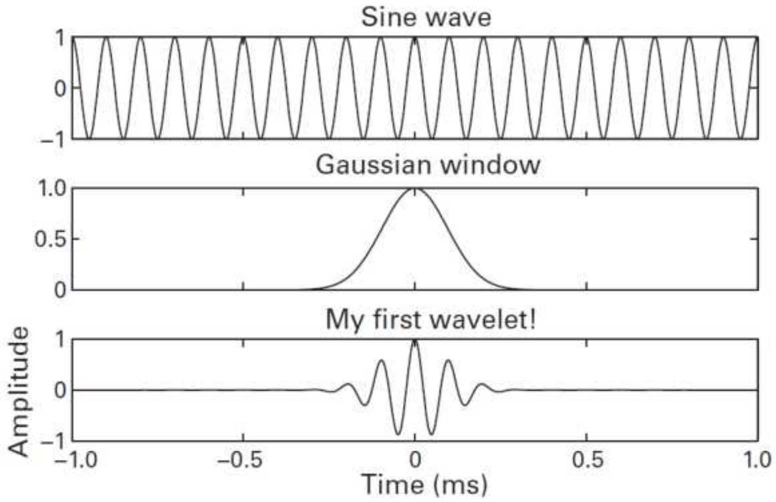
short window



long window

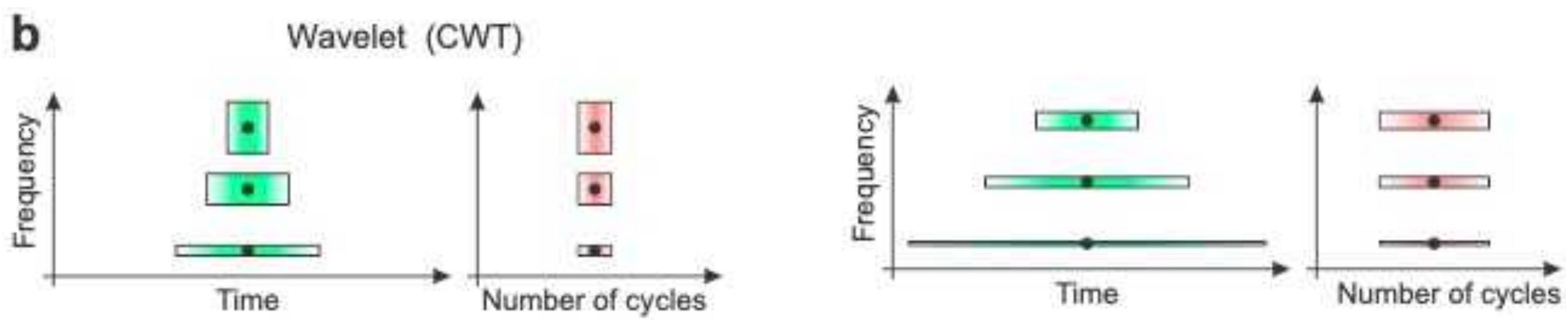
Solutions: Wavelets

Uses **variable-length windows** (short for high freq, long for low freq)
- Create wavelets by convoluting sine wave for each frequency with gaussian



Adjustable number of cycles:
- Fewer cycles: precise in time, blurred in frequency
- More cycles: precise in frequency, blurred in time

→ Provides multiresolution time–frequency analysis
- Solution to trade-off in STFT between temporal and spectral resolution



few cycles **more cycles**

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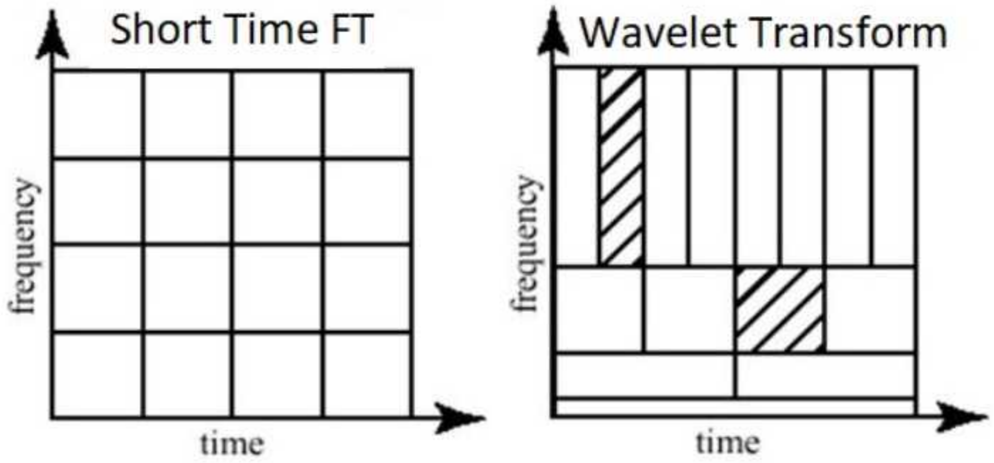

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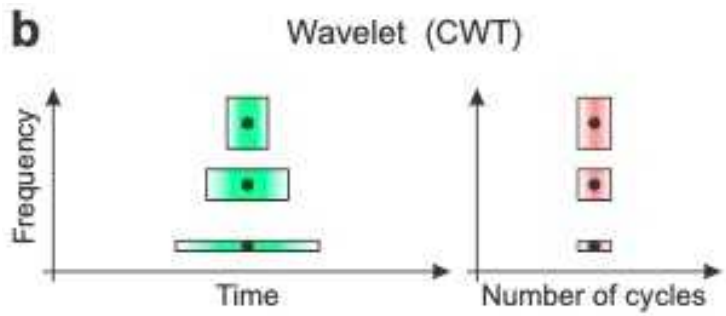
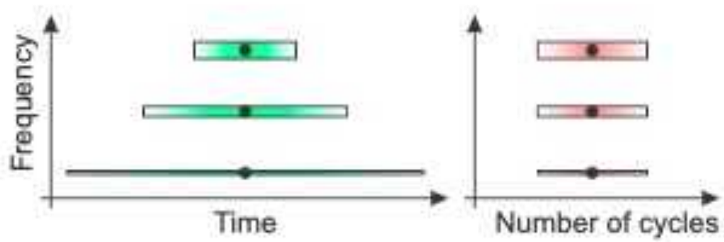


few cycles



more cycles

Analysing Neural Time Series

Problems with the Fourier Transform

Solutions: STFT/Wavelets

Heisenberg–Gabor Uncertainty Principle

Superlets

Principle

Evaluation

Conclusion

Heisenberg–Gabor Uncertainty Principle

Definition: Finite oscillation transients are difficult to localize simultaneously in both time and frequency

- Fundamental limit: can't optimise time and frequency precision simultaneously
  - STFT: length of time windows
  - Wavelets: number of cycles

  → choose either temporal precision OR frequency precision

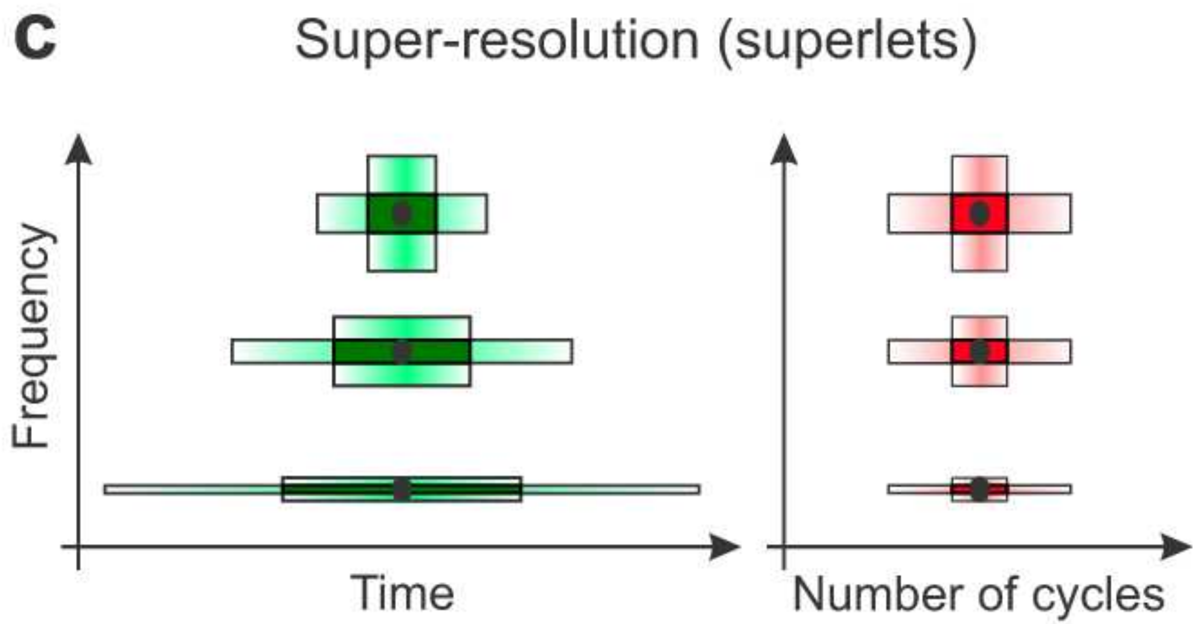
Usually: pick temporal/freq resolution due to research question

BUT NOW: SUPERLETS

Superlets

Idea: **combine wavelets with different bandwidths** (different cycle numbers) at the same frequency

Goal: improve localisation in both time and frequency simultaneously



Superlets

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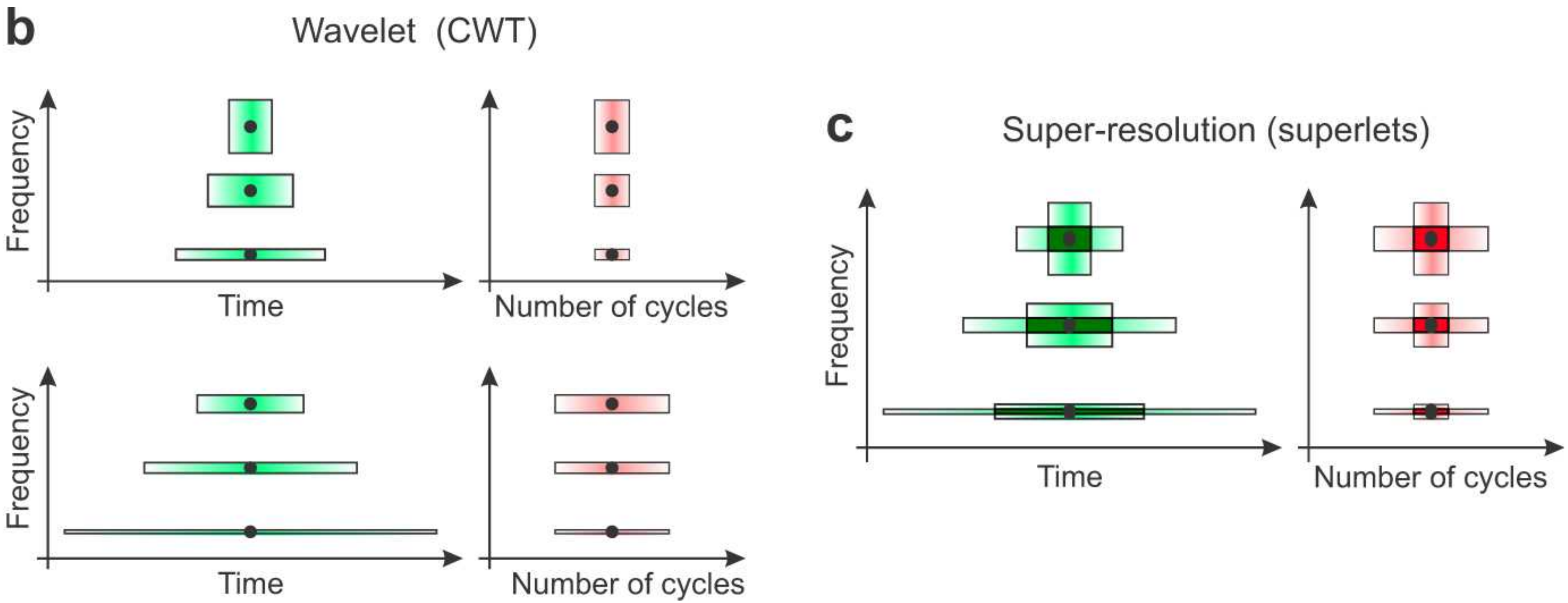
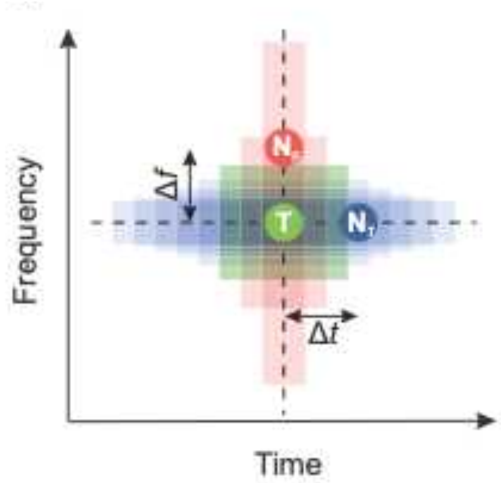


Superlet Principle

«[..] **neighbouring** high frequencies cannot be distinguished, i.e. the representation is **redundant** across wavelets [..]»

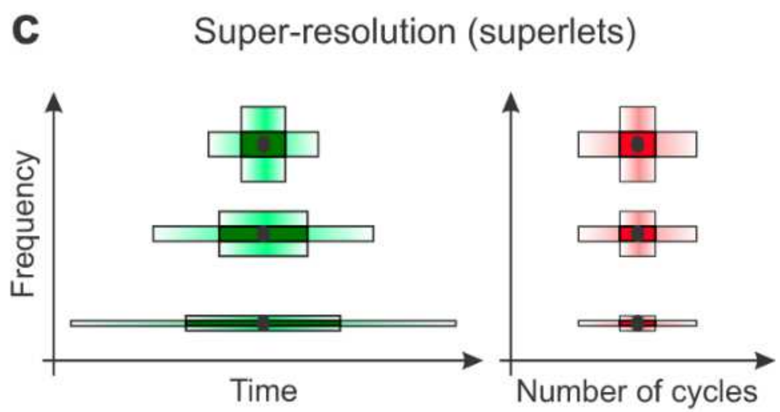


How to create a superlet for a given frequency:

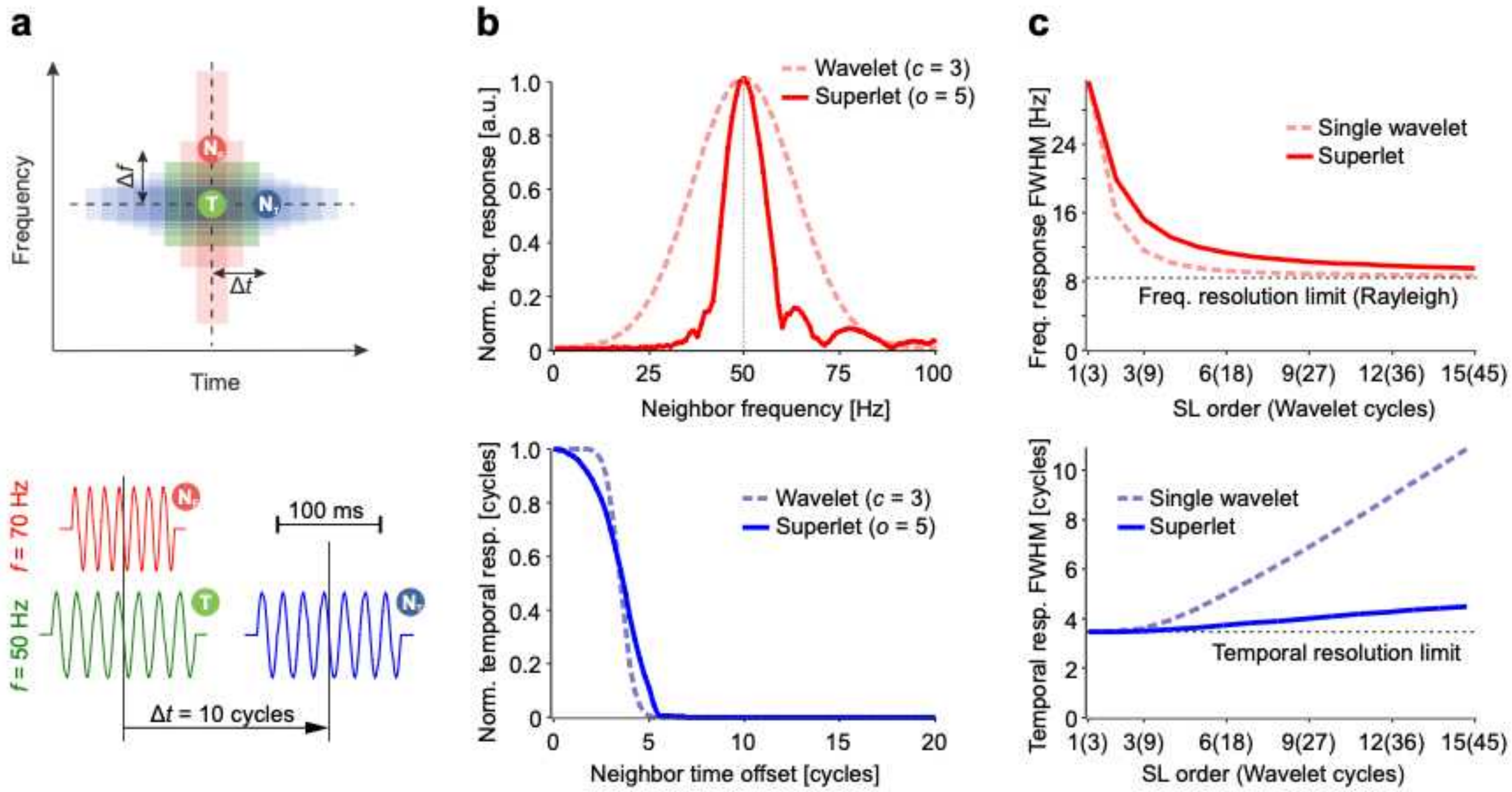
- Compute **responses of several wavelets** (order) with **increasing cycles** (starting at c_1)
- Combine responses using the **geometric mean**

  → Strong response only if both time-precise and frequency-precise wavelets agree

Effect: sharper peaks, less redundancy in neighbouring frequencies or timepoints, better separation of transients



Superlet Principle



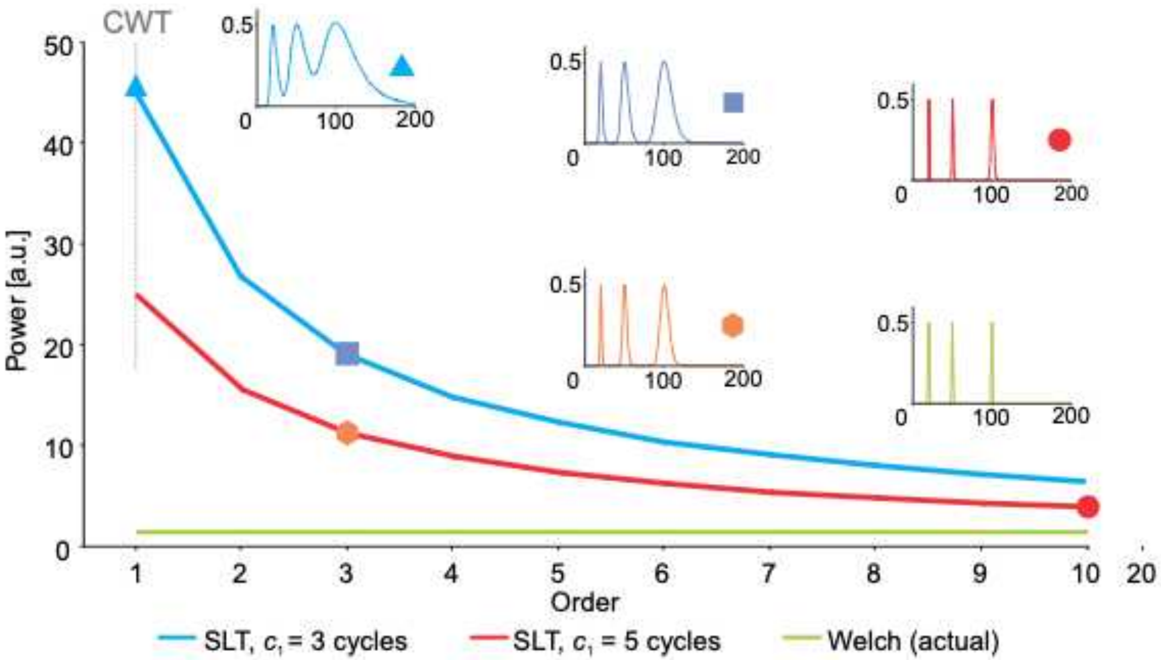
Superlet Principle

As the order increases, power estimates from the superlet drop and converge towards the true (Welch) value

At order 1 (basically normal wavelet): The power is overestimated and **wavelets smear energy in frequency**, so neighbouring high frequencies overlap

Combining more wavelets (geometric mean) **suppresses redundant responses**

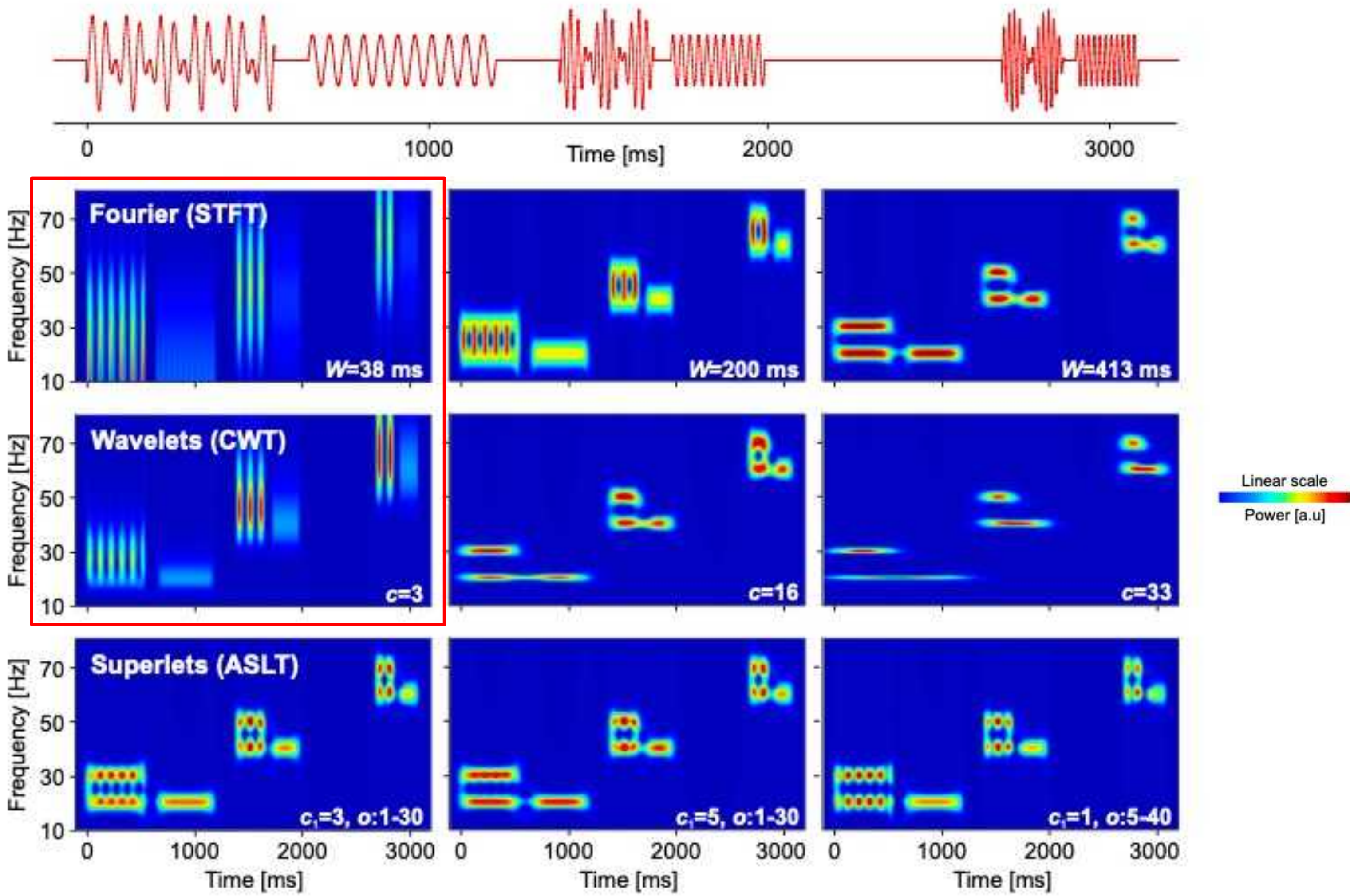
Result: The estimated power becomes more accurate and less inflated



Superlet Evaluation: Simulated Data

Test signal: 20, 40 & 60 Hz
- Neighbour in frequency (+10Hz)
- Neighbour in time (+12 cylces)

Short window: 38ms / 3 cycles
- High temporal resolution
- Smearing in frequencies

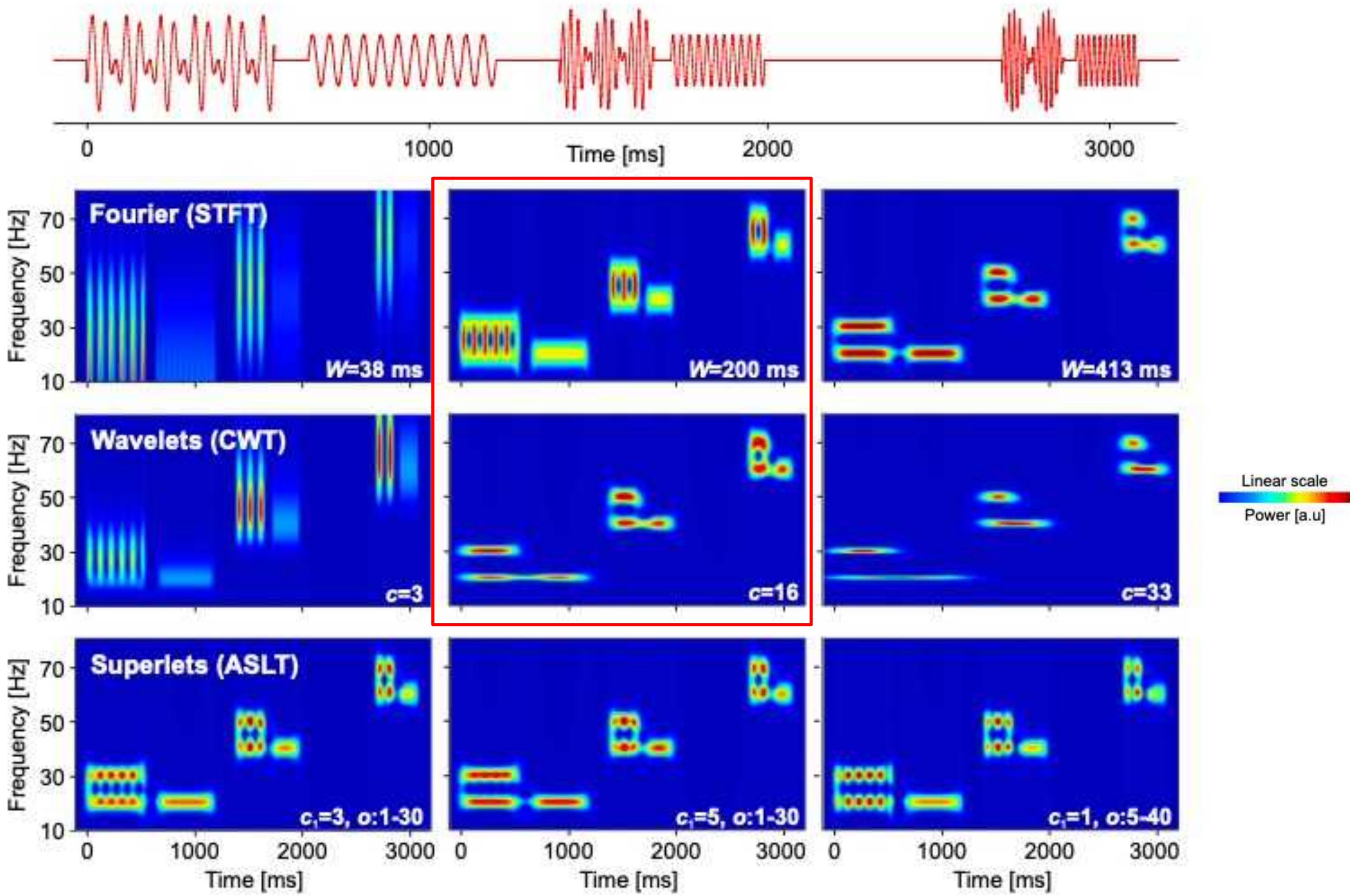


Superlet Evaluation: Simulated Data

Test signal: 20, 40 & 60 Hz
- Neighbour in frequency (+10Hz)
- Neighbour in time (+12 cylces)

Short window: 38ms / 3 cycles
- High temporal resolution
- Smearing in frequencies

Middle window: 200ms / 16 cycles
- Medium temporal resolution
- Medium frequency resolution



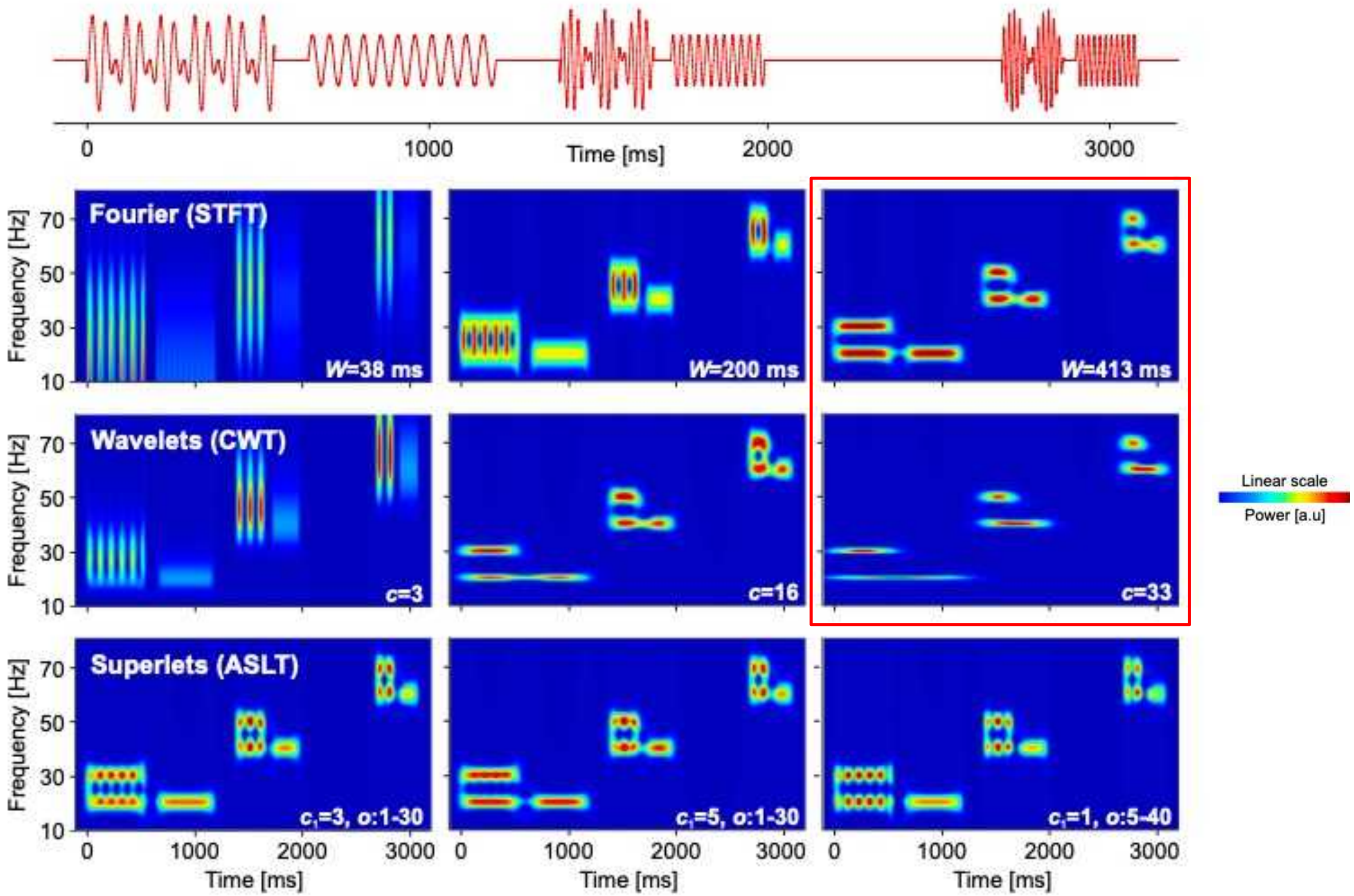
Superlet Evaluation: Simulated Data

Test signal: 20, 40 & 60 Hz
- Neighbour in frequency (+10Hz)
- Neighbour in time (+12 cylces)

Short window: 38ms / 3 cycles
- High temporal resolution
- Smearing in frequencies

Middle window: 200ms / 16 cycles
- Medium temporal resolution
- Medium frequency resolution

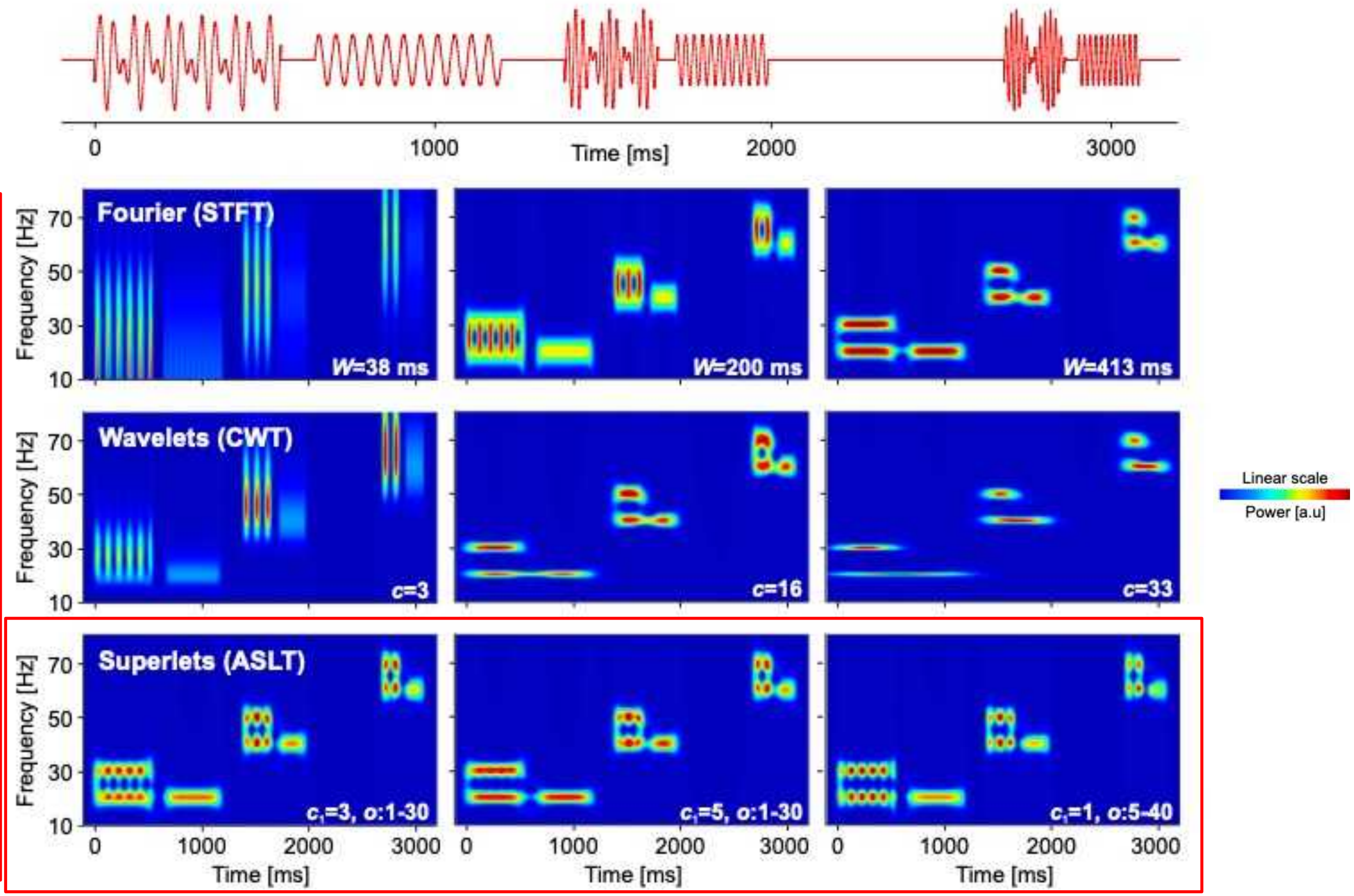
Long window: 413ms / 33 cycles
- Smearing in time
- High frequency resolution



Superlet Evaluation: Simulated Data

Test signal: 20, 40 & 60 Hz
- Neighbour in frequency (+10Hz)
- Neighbour in time (+12 cylces)

Superlets: base cycle c_1, order o
- Left (c_1=3, o=1–30):
  - short base wavelet
  - very good temporal precision, spectral peaks already sharper than wavelets
- Middle (c_1=5, o=1–30):
  - slightly longer base
  - better frequency resolution, still good time localisation
- Right (c_1=1, o=5–40):
  - very short base but very high order
  - maximises time precision and still sharp frequency separation

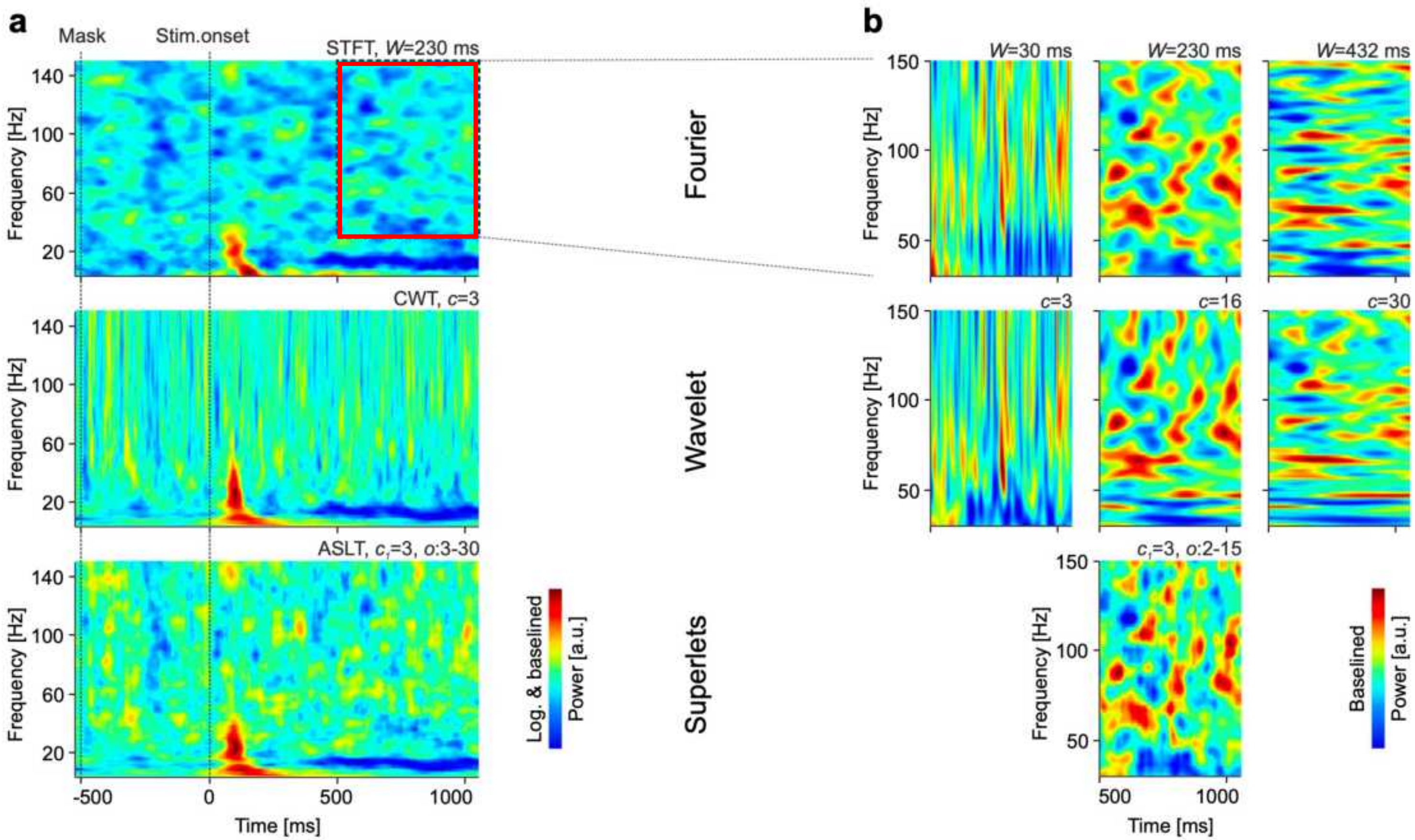


Superlet Evaluation: Real EEG Data

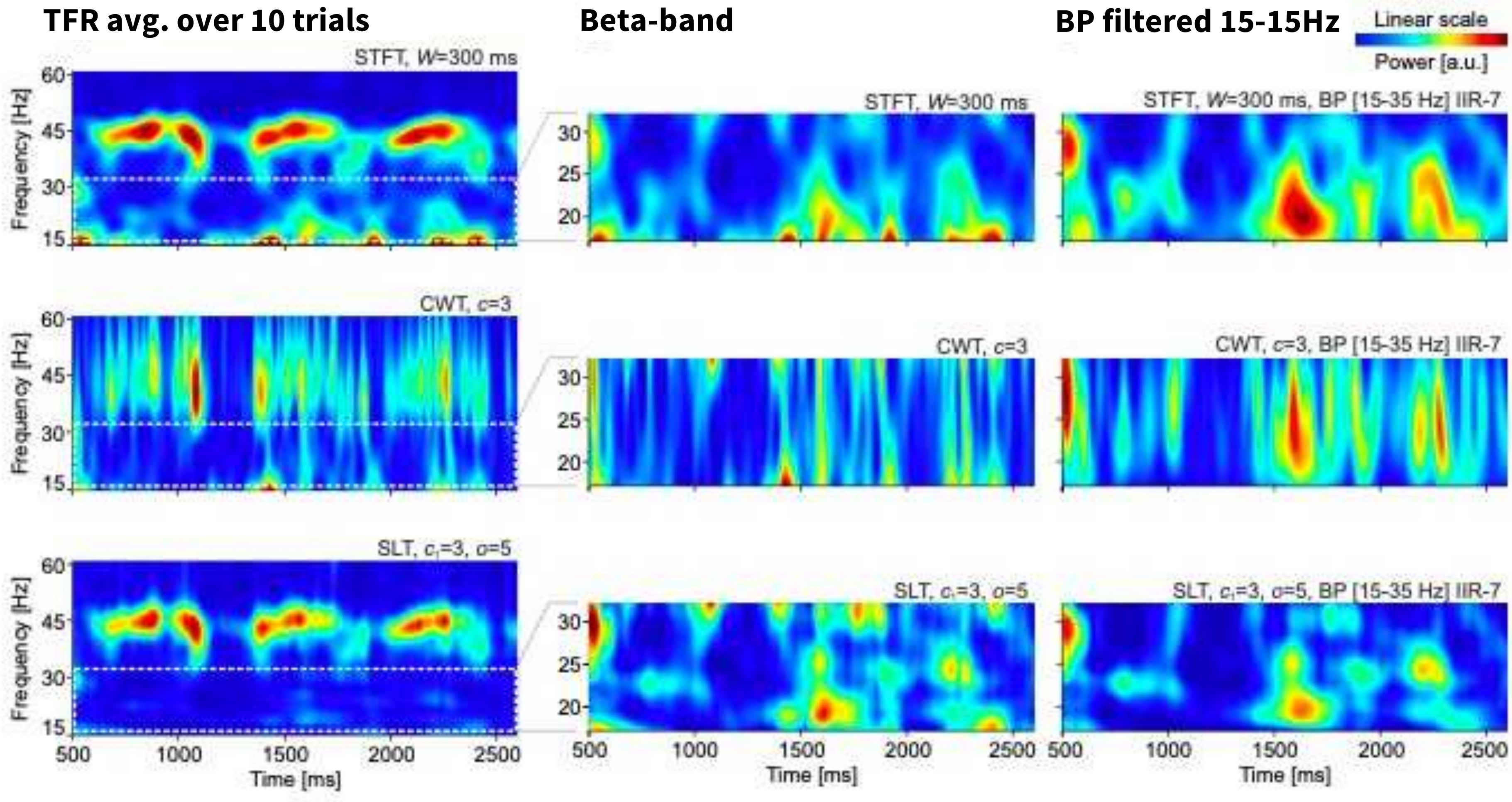
STFT: 230ms
- Smooth but blurred in time
- Misses brief high-frequency bursts

Wavelets: 3 cycles
- Captures temporal dynamics but poor frequency localisation
- Vertical smearing (broad spectral bands)

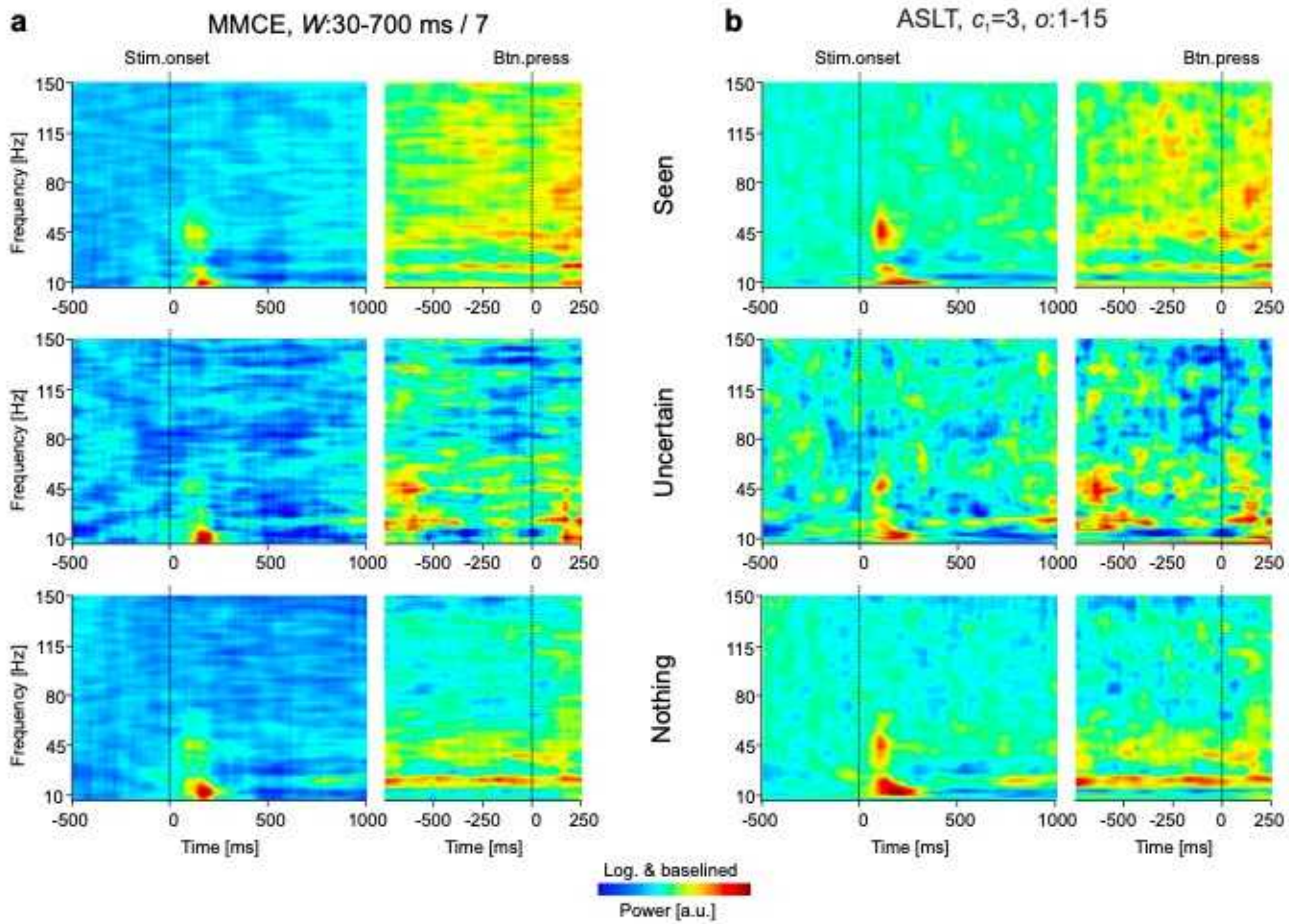
Superlets: $c_1 = 3$, o = 3-30
- Reveals sharp gamma bursts



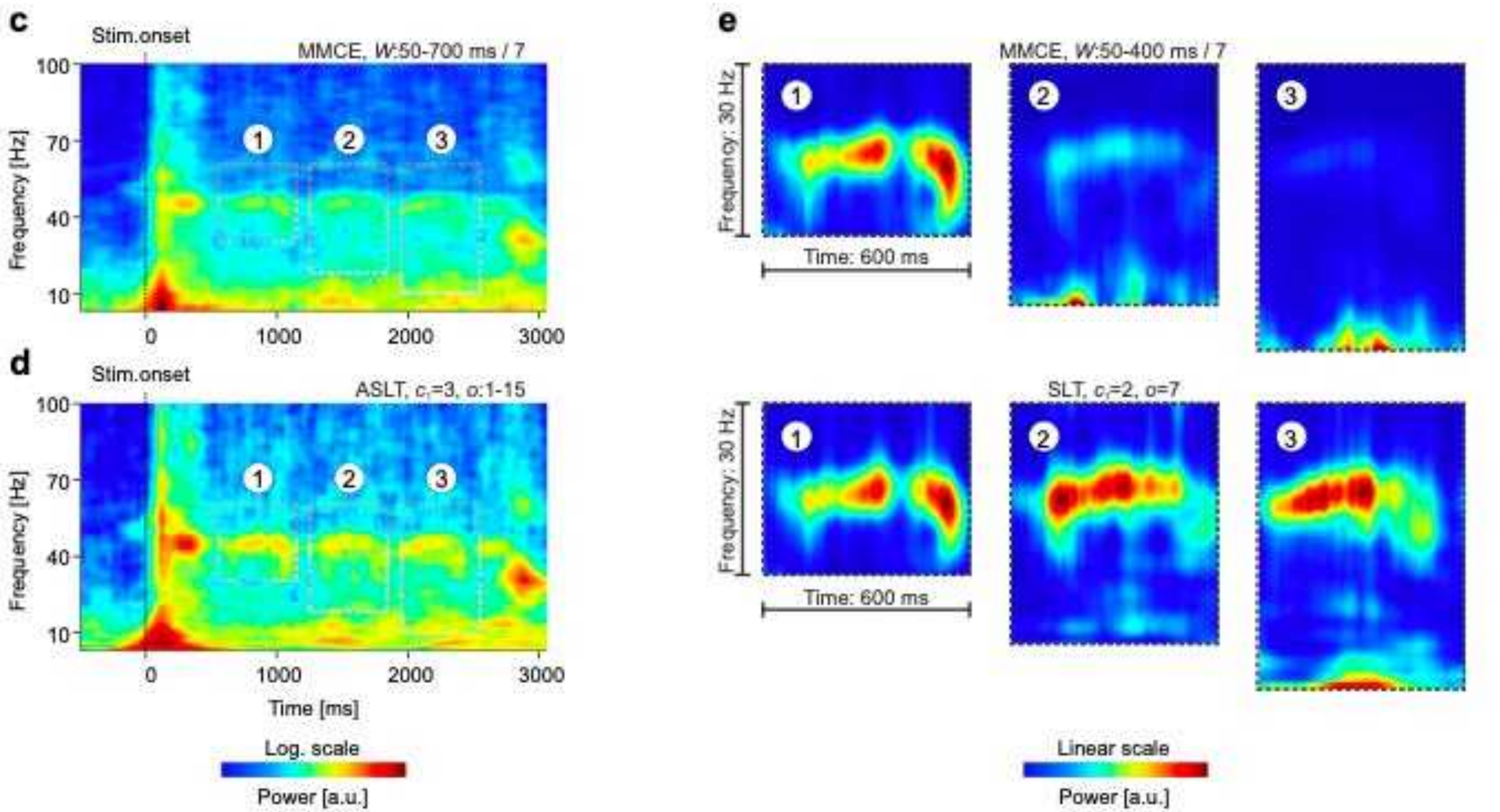
Superlet Evaluation: Averaging Time-Frequency Spectra



Superlet Evaluation: Minimum Mean Cross-Entropy (MMCE)

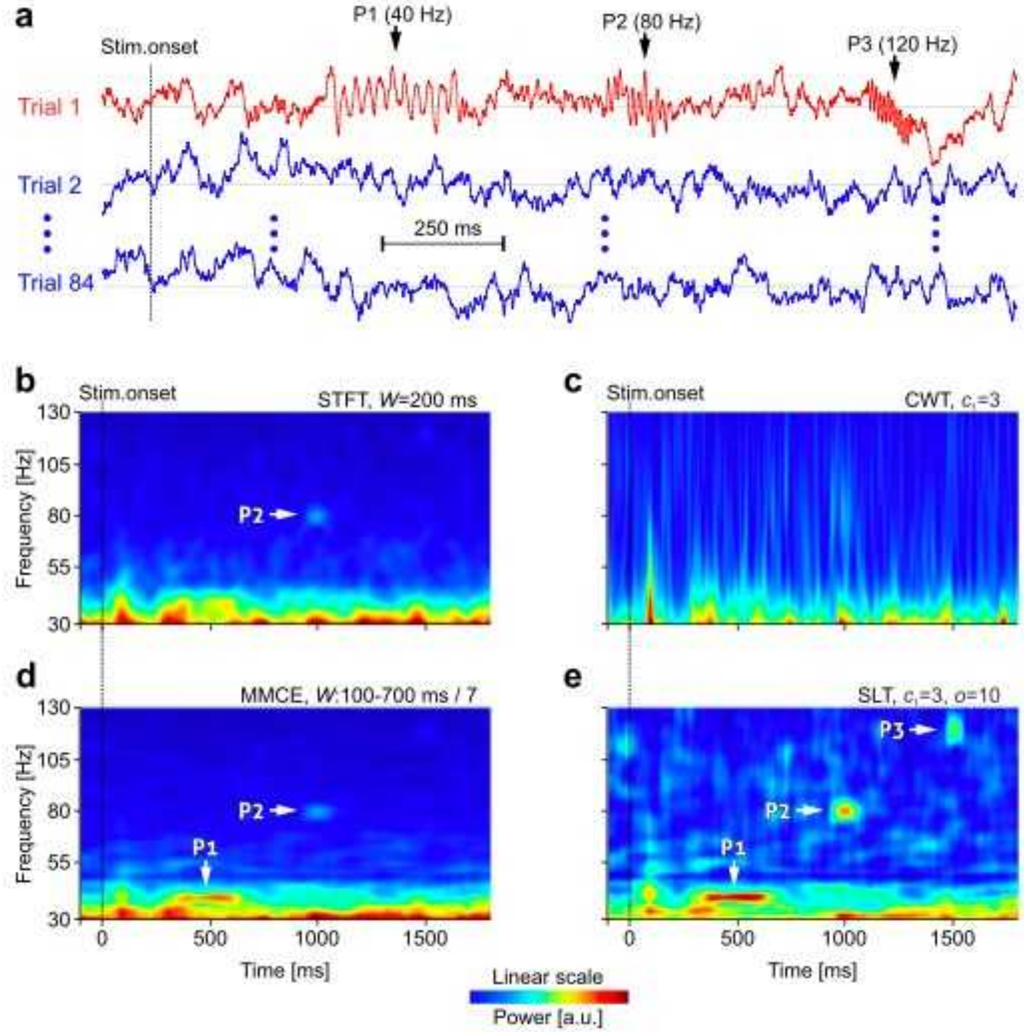


Superlet Evaluation: Minimum Mean Cross-Entropy (MMCE)

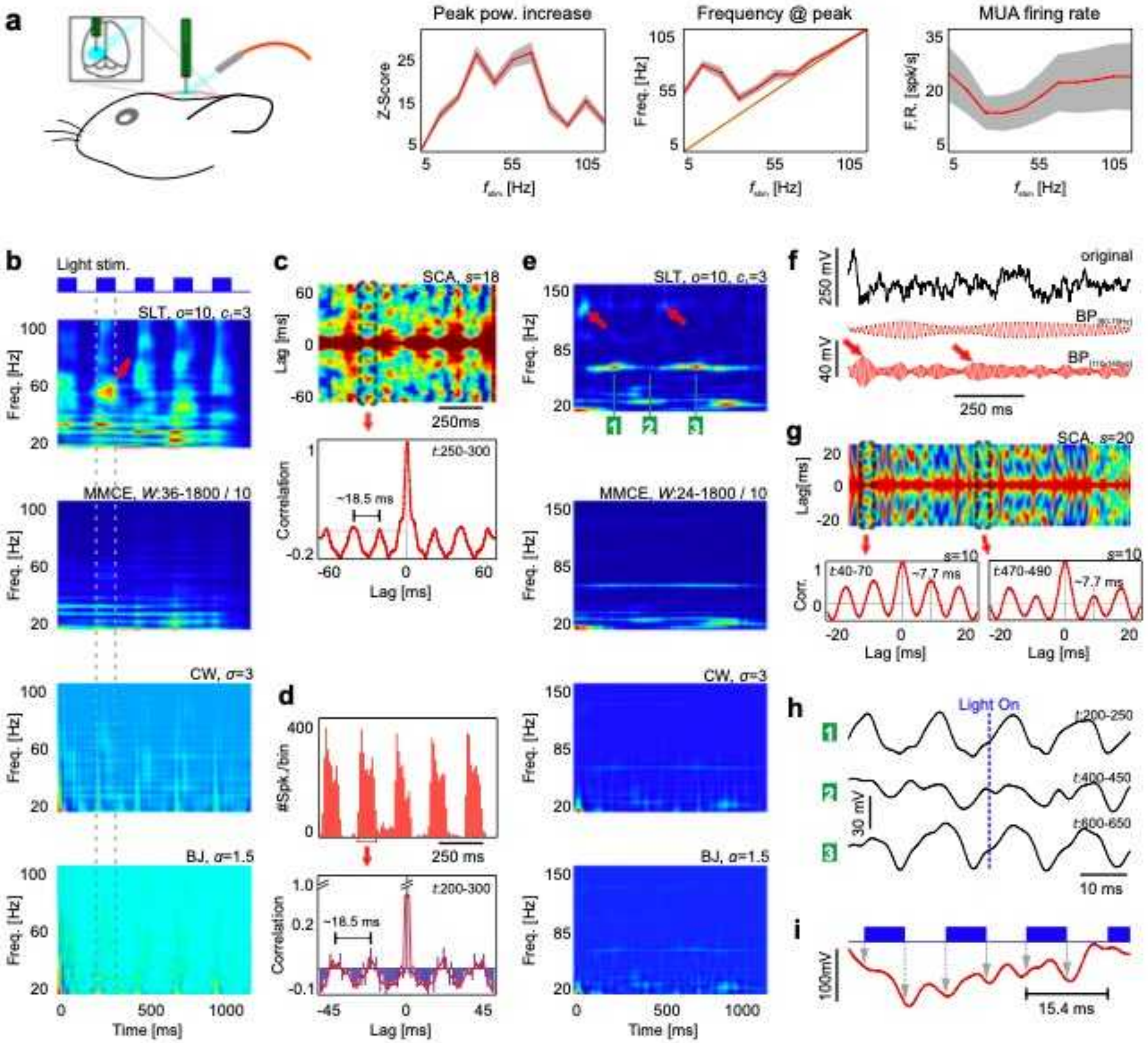


Superlet Evaluation: Single-Trial Gamma-Bursts

Superlets greates strength:
reveal brief, high-frequency (e.g. gamma) events in single trials



Superlet Evaluation: Optogenetics



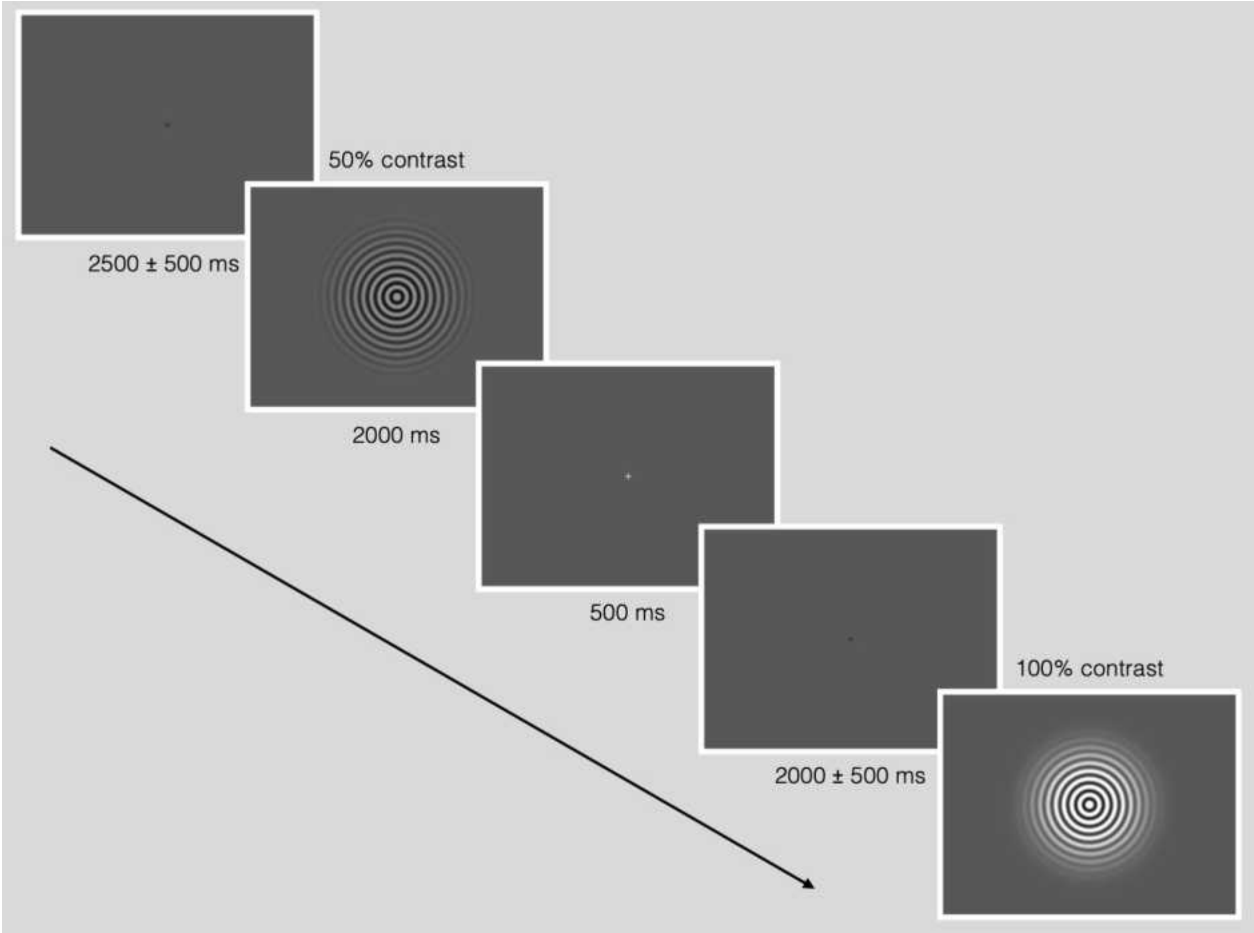
Conclusion

Conclusion

- **Sharper frequency resolution** → narrow peaks are better separated from neighbouring frequencies
- Reduced redundancy → power is not smeared into adjacent frequency bins, so the **true peak** stands out
- Improved sensitivity → small, **subject-specific peaks** become visible that wavelets or STFT might blur out
- Consistent across trials → transient bursts align better in the time–frequency map, reinforcing the peak

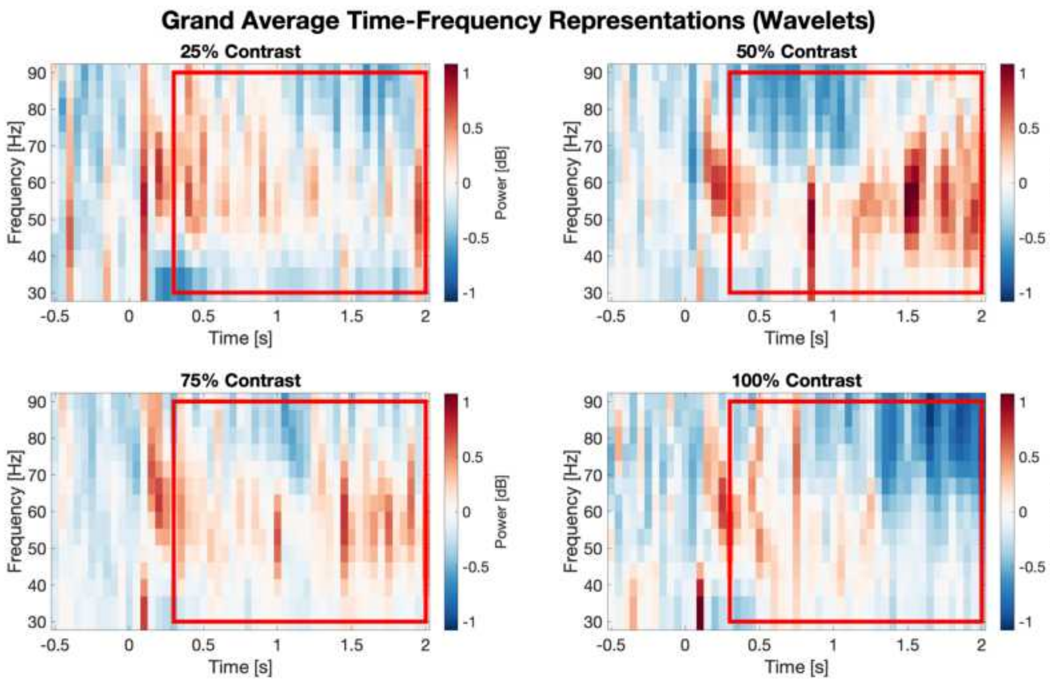
→ More accurate estimation of individual gamma frequency (IGF) at single-subject level
→ BUT: geometric mean can make sustained oscillatory activity appear more fragmented or burst-like than it truly is

Why this Method?

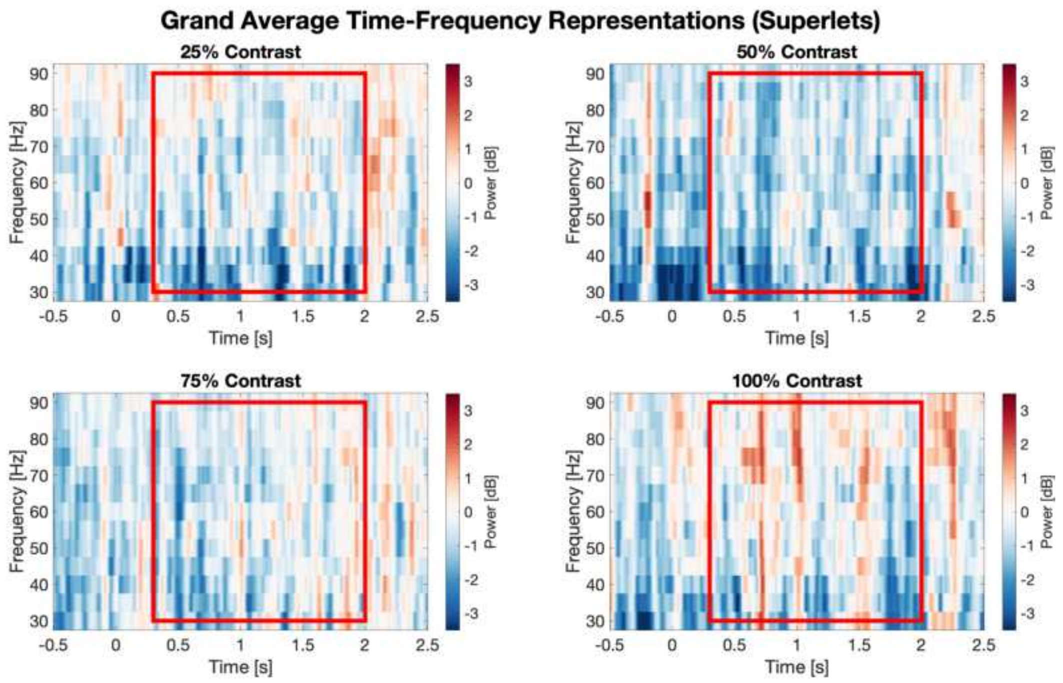


Why this Method? Gamma Peak Frequency!

→ 10 Pilot participants

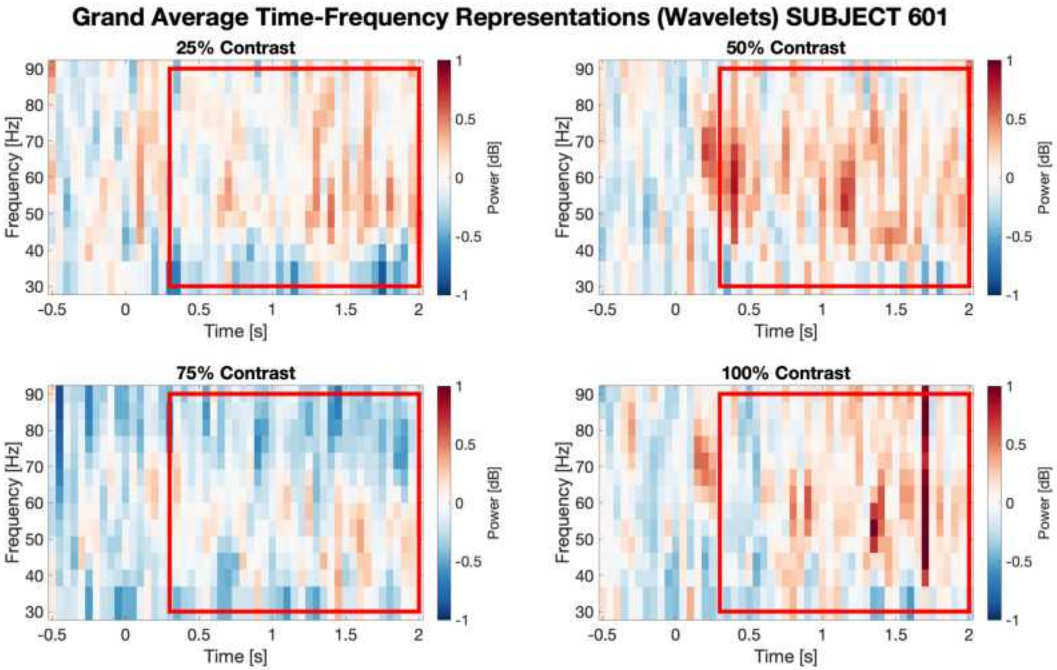


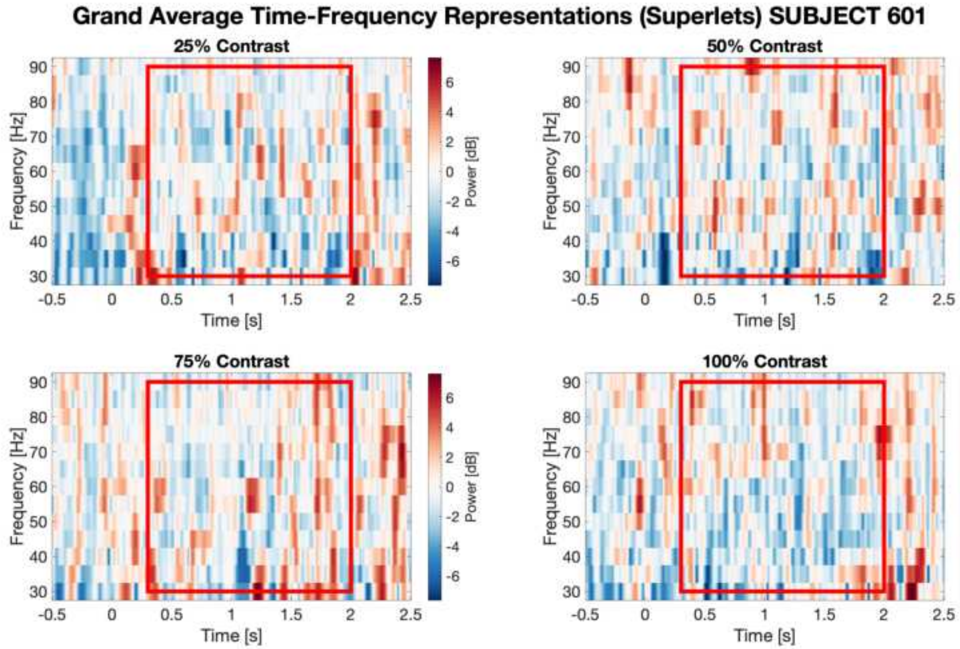
Baselined Wavelets



Baselined Superlets

Why this Method? INDIVIDUAL Gamma Peak Frequency!





Baselined Wavelets

Baselined Superlets

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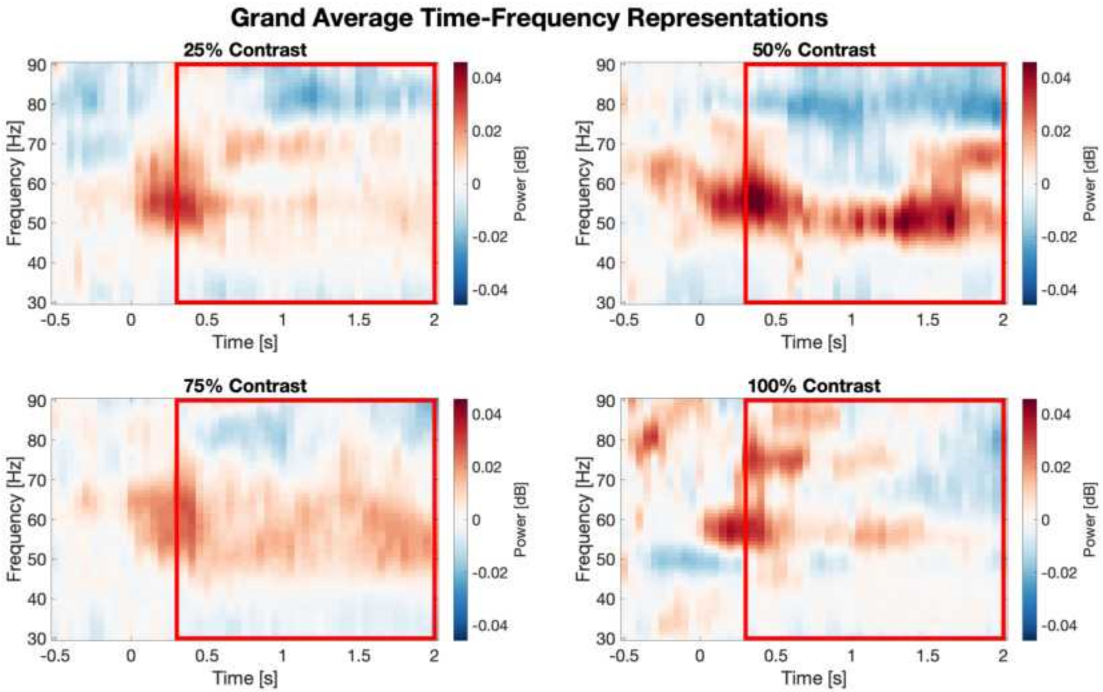

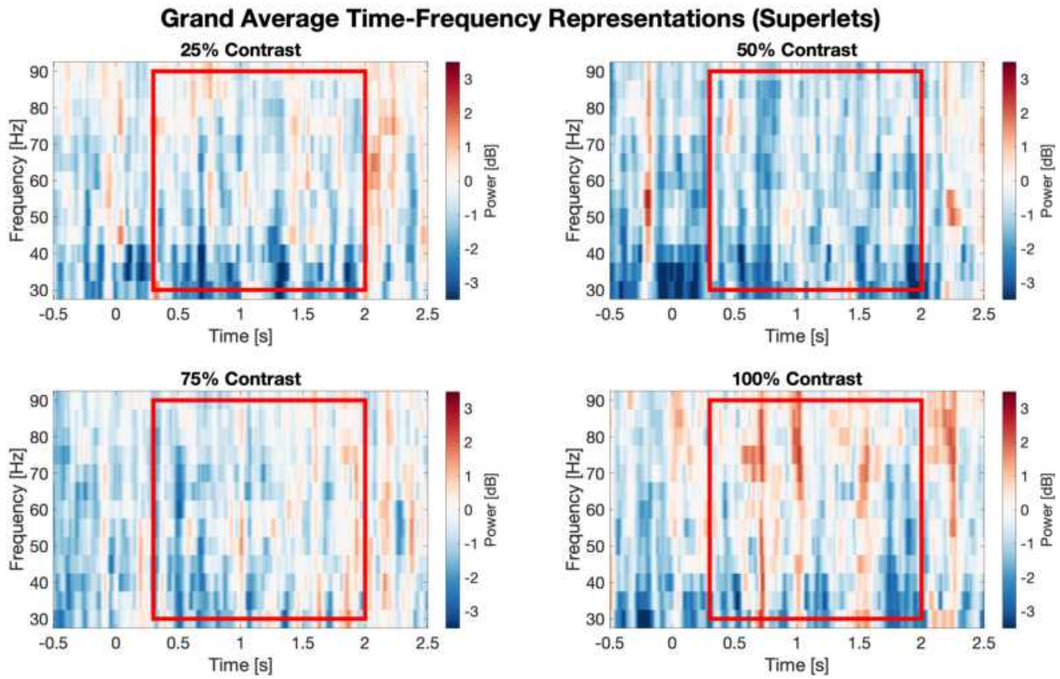

Baselined & FOOOFed multitaper STFT

Baselined Superlets